# I M O

## SOF INTERNATIONAL MATHEMATICS OLYMPIAD 2019-20

## CLASS 11

**QUESTION PAPER SET B**

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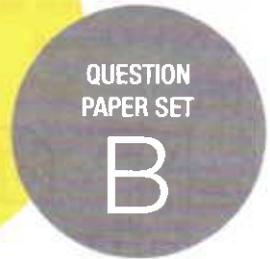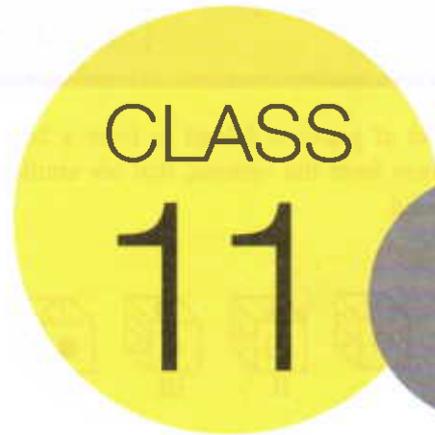
## DO NOT OPEN THIS BOOKLET UNTIL ASKED TO DO SO

### Total Questions: 50 | Time: 1 hr.

Name:..........................................................................................................................

Section:............ SOF Olympiad Roll No.:............................................. Contact No.:................................................

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## Guidelines for the Candidate

1. You will get additional ten minutes to fill up information about yourself on the OMR Sheet, before the start of the exam.

2. Write your **Name, School Code, Class, Section, Roll No.** and **Mobile Number** clearly on the **OMR Sheet** and do not forget to sign it. We will share your marks / result and other information related to SOF exams on your mobile number.

3. The Question Paper comprises four sections:

   **Logical Reasoning** (15 Questions), **Mathematical Reasoning** (20 Questions), **Everyday Mathematics** (10 Questions) and **Achievers Section** (5 Questions)

   Each question in Achievers Section carries 3 marks, whereas all other questions carry one mark each.

4. All questions are compulsory. There is no negative marking. Use of calculator is not permitted.

5. There is only ONE correct answer. Choose only ONE option for an answer.

6. To mark your choice of answers by darkening the circles on the OMR Sheet, use **HB Pencil** or **Blue / Black ball point pen** only. E.g.

   Q.16: Rahul bought 4 kg 90 g of apples, 2 kg 60 g of grapes and 5 kg 300 g of mangoes. The total weight of all the fruits he bought is_____.

   A. 11.450 kg          B. 11.000 kg          C. 11.350 kg          D. 11.250 kg

   As the correct answer is option A, you must darken the circle corresponding to option A on the OMR Sheet.     16. ● Ⓑ Ⓒ Ⓓ

7. Rough work should be done in the blank space provided in the booklet.

8. Return the OMR Sheet to the invigilator at the end of the exam.

9. Please fill in your personal details in the space provided on this page before attempting the paper.

1. The given sheet of paper is folded to form a box. Choose the boxes from the options, that are similar to the box formed.
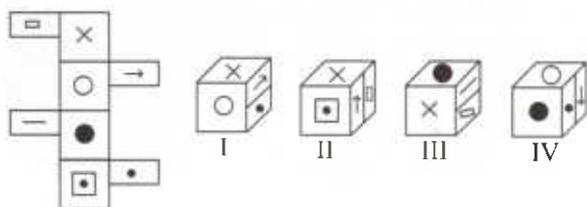


A. I, II and III
B. I, III and IV
C. I and IV
D. None of these

2. Five cities P, Q, R, S and T are connected by different modes of transport as follows.

  • P and Q are connected by boat as well as rail.
  • S and R are connected by bus and boat.
  • Q and T are connected by air only.
  • P and R are connected by boat only.
  • T and R are connected by rail and bus.

  Which mode of transport would help one to reach R starting from Q without changing the mode of transport?

A. Boat
B. Rail
C. Bus
D. Air

3. Given question consists of a set of four figures P, Q, R and S showing a sequence of folding of a piece of paper. Fig. (S) shows the manner in which the folded paper has been cut. Select a figure from the options which would most closely resemble the unfolded form of Fig. (S).
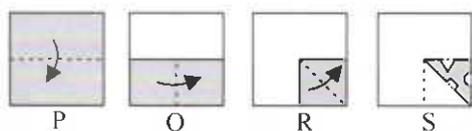


4. How many such pairs of letters are there in the word COMMUNICATION, each of which has number of letters between them in the word one less than the number of letters between them in English alphabetical series?

A. Five
B. Four
C. Six
D. More than six

5. There is a certain relationship between figures (1) and (2). Establish a similar relationship between figures (3) and (4) by selecting a suitable figure from the options that would replace the (?) in fig. (3).
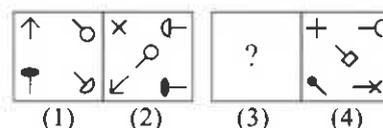


6. P, Q, R, S, T, U, V and W are sitting in a circle facing the centre. R sits third to the left of U. Only one person sits between Q and U. P sits second to the right of W. W is not an immediate neighbour of U. S is not an immediate neighbour of U. T is an immediate neighbour of W.
Who sits third to the right of V?

A. P
B. W
C. T
D. R

7. Which two figures satisfies the same conditions of placement of dots?
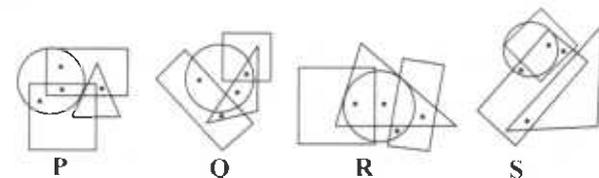


A. Q and S only
B. Q and R only
C. P and Q only
D. R and S only

8. Study the given Venn diagram carefully and answer the following question.



○ — Politicians
△ — Leaders
▢ — Unemployed
▭ — Indians
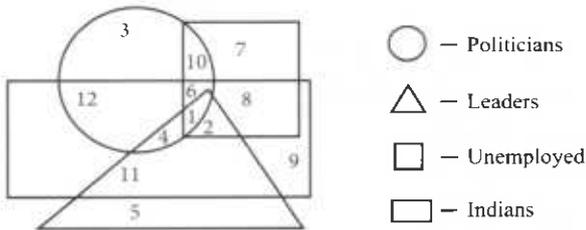
Which of the following represents the unemployed Indian leaders who are not politicians?

A.  4            B.  2
C.  1            D.  6

9. A word and number arrangement machine when given an input line of words and numbers rearranges them following a particular rule in each step. The following is an illustration of input and rearrangement.

**Input**     : 49 zone car sail 87 tower 71 56
**Step I**    : 49 car zone sail 87 tower 71 56
**Step II**   : 49 car 56 zone sail 87 tower 71
**Step III**  : 49 car 56 sail zone 87 tower 71
**Step IV**   : 49 car 56 sail 71 zone 87 tower
**Step V**    : 49 car 56 sail 71 tower zone 87
**Step VI**   : 49 car 56 sail 71 tower 87 zone

And Step VI is the last step.

As per the rules followed in the above steps, which step will be the last step for an input whose first step is "39 over 84 56 station more 41 hurry"?

A.  VI
B.  IV
C.  V
D.  VII

10. The following digits are coded as follows:

| Digit/Symbol | 6 | 7 | 1 | 2 | 8 | 3 | 4 | 9 | 5 | 0 |
|---|---|---|---|---|---|---|---|---|---|---|
| Letter | P | A | £ | J | T | ★ | O | © | W | @ |

While coding the given number following conditions are also to be observed.

**Conditions :**
(i)   If the first digit is even and the last digit is odd, then their codes are to be interchanged.
(ii)  If the first digit is odd and the last digit is even, then both are to be coded as the code for the last digit.
(iii) If both the first and the last digits are either odd or even, then both are to be coded as '#'.

4 6 3 8 5 9 2

A.  # P ★ T W © J
B.  J P ★ T W © J
C.  J P ★ T W © O
D.  # P ★ T W © #

11. Find the correct water image of the given figure.
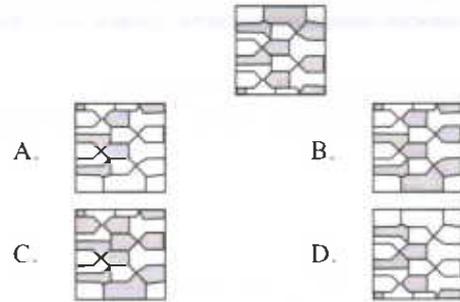


12. In which of the following figures given figure is exactly embedded as one of its parts?
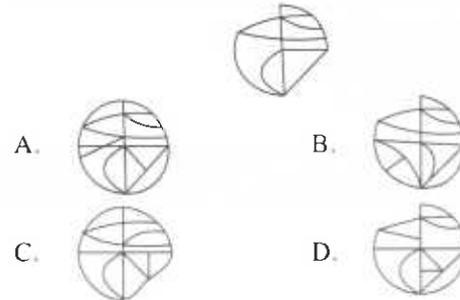


13. Find the missing number, if a certain rule is followed either row-wise or column-wise.

| 6 | 6 | 9 |
|---|---|---|
| 8 | 6 | 13 |
| 4 | ? | 3 |
| 32 | 18 | 40 |

A.  6
B.  2
C.  1
D.  4

14. If '↑' stands for '+', '↓' stands for '−', '→' stands for '÷', '←' stands for '×', '#' stands for '=', '@' stands for '>' and '©' stands for '<', then which of the following options will be incorrect?

A.  25 ↓ 12 ↑ 14 ← 4 → 6 © 23
B.  30 → 6 ↑ 5 ← 4 → 2 # 27
C.  7 ↑ 10 → 5 ← 6 ↓ 4 @ 12
D.  None of these

15. Select a figure from the options which will continue the same series as established by the Problem Figures.

Problem Figures
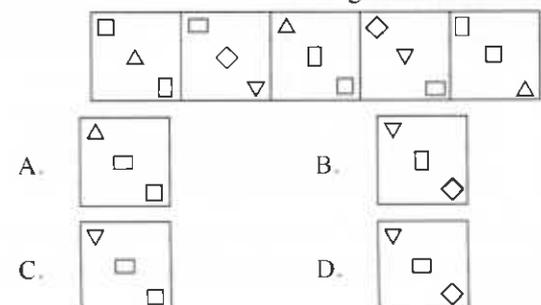
16. For any three sets $A$, $B$ and $C$ the set
$(A \cup B \cup C) \cap (A \cap B' \cap C')' \cap C'$ is equal to
    A. $B \cap C'$
    B. $B' \cap C'$
    C. $B \cap C$
    D. $A \cap B \cap C$

17. The value of $\alpha^{-n} + \alpha^{2n}$, $n \in N$ and $\alpha$ is a non-real cube root of unity, is
    A. 3, if $n$ is a multiple of 3
    B. $-1$, if $n$ is a multiple of 3
    C. 2, if $n$ is a multiple of 3
    D. None of these

18. How many words can be formed from the letters of the word DOGMATIC, if all the vowels remain together?
    A. 4140
    B. 4320
    C. 432
    D. 43

19. The mean of $n$ terms is $\bar{x}$. If the first term is increased by 1, second by 2 and so on, then the new mean is
    A. $\bar{x} + n$
    B. $\bar{x} + \dfrac{n}{2}$
    C. $\bar{x} + \dfrac{n+1}{2}$
    D. None of these

20. If the foci of the ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{b^2} = 1$ and the hyperbola
$\dfrac{x^2}{144} - \dfrac{y^2}{81} = \dfrac{1}{25}$ coincide, then the value of $b^2$ is
    A. 1
    B. 5
    C. 7
    D. 9

21. Read the given statements carefully and select the correct option.

    **Statement-1 :** $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is a contradiction.
    **Statement-2 :** $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ is a tautology.
    A. Both Statement-1 and Statement-2 are true.
    B. Both Statement-1 and Statement-2 are false.
    C. Statement-1 is true but Statement-2 is false.
    D. Statement-1 is false but Statement-2 is true.

22. Find the values of $m$ and $n$ respectively so that the given system of linear equations have infinite number of solutions.

$(2m - 1)x + 3y - 5 = 0$
$3x + (n - 1)y - 2 = 0$
    A. $\dfrac{11}{5}, \dfrac{17}{4}$
    B. $\dfrac{17}{5}, \dfrac{11}{4}$
    C. $\dfrac{11}{4}, \dfrac{17}{5}$
    D. $\dfrac{17}{4}, \dfrac{11}{5}$

23. If the $3^{rd}$ and $9^{th}$ terms of an A.P. are 4 and $-8$ respectively, then which term of this A.P. is zero?
    A. $8^{th}$
    B. $6^{th}$
    C. $5^{th}$
    D. $4^{th}$

24. $\lim\limits_{x \to 2} \dfrac{\sqrt{1+\sqrt{2+x}} - \sqrt{3}}{x-2}$ is equal to
    A. $\dfrac{1}{8\sqrt{3}}$
    B. $\dfrac{1}{\sqrt{3}}$
    C. $8\sqrt{3}$
    D. $\sqrt{3}$

25. Solution of $|2x - 3| < |x + 2|$ is
    A. $\left(-\infty, \dfrac{1}{3}\right)$
    B. $\left(\dfrac{1}{3}, 5\right)$
    C. $(5, \infty)$
    D. $\left(-\infty, \dfrac{1}{3}\right) \cup (5, \infty)$

26. A bucket is 18 cm in diameter at the top and 6 cm in diameter at the bottom. If it is 8 cm high, then find the area of sheet used in making the bucket.
    A. $134 \pi$ cm$^2$
    B. $109 \pi$ cm$^2$
    C. $121 \pi$ cm$^2$
    D. $129 \pi$ cm$^2$

27. A, B and C take one shot each at a target. Their probabilities of hitting the target are respectively 0.4, 0.5 and 0.8. The probability that at least two of them hit the target, is

A. $\dfrac{3}{5}$

B. $\dfrac{13}{25}$

C. $\dfrac{9}{25}$

D. None of these
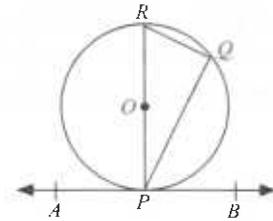
28. The domain of the function $f(x) = \log_x \cos x$ is

A. $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right) - \{1\}$

B. $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right] - \{1\}$

C. $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

D. None of these

29. If $p_1$ and $p_2$ are the lengths of the perpendiculars from the origin to the straight lines $x \sec\theta + y \csc\theta = a$ and $x \cos\theta - y \sin\theta = a \cos 2\theta$ respectively, then the value of $4p_1^2 + p_2^2$ is

A. $4a^2$

B. $2a^2$

C. $a^2$

D. None of these

30. If $C_0, C_1, C_2, ..., C_n$ denote the coefficients in the binomial expansion of $(1 + x)^n$, then

$C_0 + 2 \cdot C_1 + 3 \cdot C_2 + ... + (n + 1)C_n =$

A. $n \, 2^{n+1}$

B. $(n + 1)2^{n-1}$

C. $(n + 2)2^{n-1}$

D. $(n + 2) \, 2^n$

31. In the given figure (not drawn to scale), $AB$ is a tangent to the circle with centre $O$. If $\angle BPQ = 60°$, then find $\angle PRQ$.



A. 30°

B. 60°

C. 45°

D. 75°

32. The ratio in which the line joining $(-2, 4, 7)$ and $(3, -5, 8)$ is divided by $yz$-plane, is

A. 5 : 3

B. 3 : 5

C. 2 : 3

D. 2 : 5

33. If $\sin 6\theta + \sin 4\theta + \sin 2\theta = 0$, then $\theta =$

A. $\dfrac{n\pi}{2}$ or $n\pi \pm \dfrac{\pi}{2}$

B. $\dfrac{n\pi}{4}$ or $n\pi \pm \dfrac{\pi}{6}$

C. $\dfrac{n\pi}{4}$ or $2n\pi \pm \dfrac{\pi}{6}$

D. None of these

34. $P(n) : 3^{2n+2} - 8n - 9$ is divisible by 64, is true for

A. All $n \in N \cup \{0\}$

B. $n \geq 2, n \in N$

C. $n \in N, n > 2$

D. None of these

35. If twice the area of a smaller square is subtracted from the area of a larger square, the result is $14$ cm². However, if twice the area of the larger square is added to three times the area of the smaller square, the result is $203$ cm². Find the sides of both the squares.

A. 5 cm, 6 cm     B. 5 cm, 8 cm

C. 6 cm, 8 cm     D. 7 cm, 9 cm

## EVERYDAY MATHEMATICS

36. A man on the top of a tower, standing on the seashore finds that a boat coming towards him takes 10 minutes for the angle of depression to change from 30° to 60°. Find the time taken by the boat to reach the shore from this position.

A. 8 mins

B. 3 mins

C. 5 mins

D. 12 mins

37. A man is employed to count ₹ 10710. He counts at the rate of ₹ 180 per minute for half an hour. After this he counts at the rate of ₹ 3 less every minute than the preceding minute. Find the time taken by him to count the entire amount.

A. 1 hr 10 mins

B. 1 hr 39 mins

C. 1 hr 20 mins

D. 1 hr 29 mins

38. 12 men complete a work in 9 days. After they have worked for 6 days, 6 more men join them. How many days will they take to complete the remaining work?

A. 2
B. 3
C. 4
D. 5

39. Out of 30 teachers of a school, a teacher of age 60 years retired. On his place, another teacher of age 30 years was appointed, due to this, the mean age of the teachers will

A. Decrease by 6 months
B. Remains same
C. Decrease by 1 year
D. Decrease by 2 years.

40. A tradesman gives 4% discount on the marked price and gives 1 article free for buying every 15 articles and thus gain 35%. The marked price is above the cost price by

A. 20%
B. 39%
C. 40%
D. 50%

41. A boat goes 32 km upstream and 36 km downstream in 7 hrs. Again it goes 40 km upstream and 48 km downstream in 9 hrs. The speed of boat in still water is

A. 2 km/hr
B. 10 km/hr
C. 5 km/hr
D. 4 km/hr

42. A mixture contains 80% acid and rest water. Part of the mixture that should be removed and replaced by the same amount of water to make the ratio of acid and water 4 : 3 is

A. $\left(\dfrac{1}{3}\right)^{rd}$

B. $\left(\dfrac{2}{7}\right)^{th}$

C. $\left(\dfrac{2}{3}\right)^{rd}$

D. $\left(\dfrac{3}{7}\right)^{th}$

43. How many spherical bullets can be made out of a lead cylinder 15 cm high and with base radius 3 cm, if each bullet being 5 mm in diameter?

A. 6000
B. 6480
C. 7260
D. 7800

44. In a school, there are 20 teachers who teach mathematics or physics. Of these, 12 teach mathematics and 4 teachers teach both mathematics and physics. The number of teachers teaching only physics are

A. 4
B. 12
C. 8
D. 16

45. The price of a certain item is increased by 15%. If a consumer wants to keep his expenditure on the item same as before, then how much percent must he reduce his consumption of that item ?

A. $10\dfrac{20}{23}\%$

B. $13\dfrac{1}{23}\%$

C. $16\dfrac{2}{3}\%$

D. 15%

## ACHIEVERS SECTION

46. If $A$ and $B$ are two events such that $P(A) = \dfrac{1}{2}$ and $P(B) = \dfrac{2}{3}$, then which of the following options is incorrect?

A. $P(A \cup B) \geq \dfrac{2}{3}$

B. $P(A \cap \bar{B}) > \dfrac{1}{3}$

C. $\dfrac{1}{6} \leq P(A \cap B) \leq \dfrac{1}{2}$

D. $\dfrac{1}{6} \leq P(\bar{A} \cap B) \leq \dfrac{1}{2}$

47. Let $a$, $b$, $c$, $p$, $q$ be real numbers. Suppose $\alpha$, $\beta$ are the roots of the equation $x^2 + 2px + q = 0$ and $\alpha$, $\dfrac{1}{\beta}$ are the roots of the equation $ax^2 + 2bx + c = 0$, where $\beta^2 \notin \{-1, 0, 1\}$.

**Statement-1** : $(p^2 - q)(b^2 - ac) \geq 0$

**Statement-2** : $b \neq pa$ or $c \neq qa$

A. Both Statement-1 and Statement-2 are true.
B. Both Statement-1 and Statement-2 are false.
C. Statement-1 is true but Statement-2 is false.
D. Statement-1 is false but Statement-2 is true.

48. Match the following columns and select the correct option.

| | Column I | | Column II |
|---|---|---|---|
| P. | If $\alpha, \beta$ are the solutions of $\sin x = -\frac{1}{2}$ in $[0, 2\pi]$ and $\alpha, \gamma$ are the solutions of $\cos x = -\frac{\sqrt{3}}{2}$ in $[0, 2\pi]$, then | (i) | $\alpha - \beta = \pi$ |
| Q. | If $\alpha, \beta$ are the solutions of $\cot x = -\sqrt{3}$ in $[0, 2\pi]$ and $\alpha, \gamma$ are the solutions of $\csc x = -2$ in $[0, 2\pi]$, then | (ii) | $\beta - \gamma = \pi$ |
| R. | If $\alpha, \beta$ are the solutions of $\sin x = -\frac{1}{2}$ in $[0, 2\pi]$ and $\alpha, \gamma$ are the solutions of $\tan x = \frac{1}{\sqrt{3}}$ in $[0, 2\pi]$, then | (iii) | $\alpha - \gamma = \pi$ |
| | | (iv) | $\alpha + \beta = 3\pi$ |
| | | (v) | $\beta + \gamma = 2\pi$ |

A. P - (ii), (iv); Q - (i), (v); R - (iii), (iv), (v)
B. P - (ii), (v); Q - (ii), (iii); R - (ii), (iii), (v)
C. P - (i), (iii); Q - (i), (iv); R - (ii), (v)
D. P - (i), (iv), (v); Q - (ii), (iii); R - (i), (iv)

49. Select the correct option.
A. If the eccentricity of an ellipse is 5/8 and the distance between its foci is 10, then its latus rectum is $\frac{39}{4}$.
B. Eccentricity of the ellipse whose latus rectum is equal to the distance between two focus points, is $\frac{\sqrt{5}-1}{2}$.
C. In an ellipse, if distance between the directrices be thrice the distance between the foci, then its eccentricity is $\frac{1}{\sqrt{3}}$.
D. All of these

50. Different words are being formed by arranging the letters of the word 'ARRANGE'. All the words obtained are written in the form of a dictionary.
(i) Find the number of words in which the two 'R' are not together.
(ii) Find the rank of the word 'ARRANGE' in the dictionary.

| | (i) | (ii) |
|---|---|---|
| A. | 1260 | 340 |
| B. | 660 | 341 |
| C. | 900 | 342 |
| D. | 240 | 343 |

**SPACE FOR ROUGH WORK**

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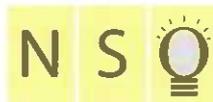
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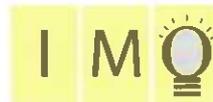
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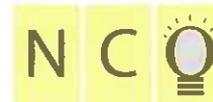
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